GEORGE MASON UNIVERSITY

Learning Through Teaching – An Online Setting

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ABSTRACT

A powerful way for students to master a subject is to engage them in teaching it to someone. This is especially true for mastering concepts in mathematics. In a traditional classroom we can give markers to students and send them to the whiteboard to solve problems. In an online course the electronic whiteboard is not quite as friendly. Though recent innovations make it possible to "write mathematics" on the electronic whiteboard, the process is quite time-consuming.

In our online pre-calculus course we supply a student with a PDF file containing the solution to a problem and ask the student to create a YouTube video in which they play the role of instructor. They are required to explain the steps involved in solving the problem and why each particular step "makes sense." We then provide a critique of their video and if needed require them to supply a revised video. Select revised videos are made available online for the entire class.

We present three of the student videos, feedback on one of the videos, and the corresponding revised video.

Class Participation for Week 8

- Student responses:
  https://youtu.be/4Q7-Ho0hmps
  https://youtu.be/YWMWv3CHK1A

6.1/44

Find $\tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right)$.

Note: $\tan^{-1}(\tan(x)) = x$ for x in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Note: $x = -\frac{2\pi}{3}$ is not in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Replace $-\frac{2\pi}{3}$ by an angle θ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\tan\theta = \tan\left(-\frac{2\pi}{3}\right)$. (Why?)

Use the referance angle of $-\frac{2\pi}{3}$ together with appropriate sign.

Tangent is positive in quadrant III, so we replace $\tan\left(-\frac{2\pi}{3}\right)$ by $\tan\left(\frac{\pi}{3}\right)$.

$\tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$.

6.1/54

$$f(x) = 2\tan x - 3$$
$$y = 2\tan x - 3$$
$$x = 2\tan y - 3$$
$$2\tan y = x + 3$$
$$\tan y = \frac{x+3}{2}$$
$$y = \tan^{-1}\left(\frac{x+3}{2}\right) = f^{-1}(x)$$

The domain of $f^{-1}$is all real numbers, or $(-\infty, \infty)$ in interval notation. (Why?)
So the range of f is all real num\bers. (Why?)

6.2/32

Find $\cot\left[\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$.

Set $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)$. Then $\cos\theta = -\frac{\sqrt{3}}{3}$. It follows that θ is in quadrant II. (Why?)

We set $x = -\sqrt{3}$ and $r = 3$. (Why?)

Class Participation for Week 12

- Student responses:
  https://youtu.be/BmKUxAglgw8
  https://youtu.be/MZFhzCBpOew
  https://youtu.be/XFnJdtCS--Y

6.5/82

Find $\tan\left(\frac{\pi}{4} - \cos^{-1}\frac{3}{5}\right)$.

Let $\alpha = \cos^{-1}\frac{3}{5}$.

α is in quadrant I. (Why?)

Then $\cos\alpha = \frac{3}{5}$, $0 \leqslant \alpha \leqslant \frac{\pi}{2}$.

$\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \sqrt{1-\cos^2\alpha}$ (Why?)

$$\sin\alpha = \sqrt{1-\cos^2\alpha}$$
$$= \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$
$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{4/5}{3/5} = \frac{4}{3}$$
$$\tan\left(\frac{\pi}{4} - \cos^{-1}\frac{3}{5}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\cos^{-1}\frac{3}{5}\right)}{1+\tan\left(\frac{\pi}{4}\right)\tan\left(\cos^{-1}\frac{3}{5}\right)}$$
$$= \frac{1-\frac{4}{3}}{1+(1)\left(\frac{4}{3}\right)} = \frac{-\frac{1}{3}}{\frac{7}{3}} = -\frac{1}{7}$$

6.6/84

Find $\tan\left(2\tan^{-1}\frac{3}{4}\right)$.

Feedback to student on class participation project

Tangent of the quantity A minus B

The arc cosine of three fifths or inverse cosine of ...

The range of the inverse cosine function is the closed interval zero to pi. So the inverse cosine of three fifths must be in quadrant I or quadrant II. Since the cosine function is positive in quadrant I and negative in quadrant II, the inverse cosine of three fifths must bean angle in quadrant I.

Now that we have found the tangent of alpha, we will find the tangent of the quantity pi over four minus alpha.

We know that the tangent of the difference of two angles equals the fraction whose numerator is the tangent of the first angle minus the tangent of the second angel and whose denominator is one minus the product of the tangent of the first angle and the tangent of the second angle.

In our problem, the first angle is pi over 4 and the second angle is alpha

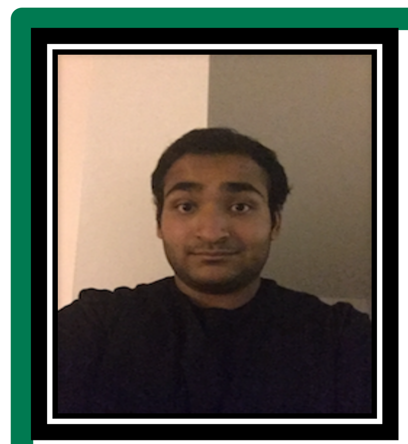
We simplify the complex fraction $\frac{\frac{3}{2}}{1-\frac{9}{16}}$ by multiplying numerator and denomintor by sixteen, the least common divisor.

Student Feedback

Hello Professor,

I think making the videos was helpful because it made you break down the problem and explain it in your own words. Everyone has their own way of understanding a math problem, so voicing it out loud for yourself slowed the process down so that I could actually understand and think about what I was doing, instead of just simply plugging in an answer. It helped me understand why I got the answer basically.

-Amber Harlow



Student Feedback

Making the videos benefitted me a lot and seemed very interactive. It's like putting math concepts and algebraic expressions into words. Rather than solving the equation, you explain how it's being solved. I felt like a teacher trying to explain how to solve a problem to a group of students. It's very stimulating to hear your own voice trying to explain a problem and how to solve it.

- Humzza Raja

What I Learned

- **I became more aware of the mathematical dialects ("their own words") used by my students when solving problems**
- **Small doses of online encouragement often result in a significant increase in student effort**
- **Watching the student videos showed me what concepts are difficult for students, even when "solutions" are provided**
- **I gained insight as to which lecture notes/course videos need to be modified to provide a more successful learning environment**